Heat Transfer in Laminar Flow with Wall Axial Conduction and External Convection

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Analytical solution is obtained for laminar forced convection inside tubes including wall conduction effects in the axial direction, based on a radially lumped wall temperature model, and accounting for external convection. The ideas in the generalized integral transform technique are extended to accommodate for the resulting more involved boundary condition and accurate numerical results obtained for quantities of practical interest such as bulk fluid temperature, lumped wall temperature, and Nusselt number. The effects of external convection and axial conduction along the wall on these heat transfer quantities are then investigated through consideration of typical values for, respectively, Biot number and a wall-to-fluid conjugation parameter. Convergence characteristics of the present approach are also briefly examined.

Nomenclature

Bi	=	Biot number with respect to fluid, $h_\infty r_2/K_f$
Bi^*	=	Biot number with respect to wall, $h_\infty r_1/K_s$
c_P	=	specific heat of the fluid
D_h	=	hydraulic diameter of circular tube, $2r_1$
$h(z)$	=	convective heat transfer coefficient (internal)
h_∞	=	convective heat transfer coefficient (external)
K_s, K_f	=	thermal conductivities of solid and fluid, respectively
$\hat{K}$	=	thermal conductivities ratio, K_f/K_s
L^*, L	=	duct length, dimensional and dimensionless, respectively
$Nu(Z)$	=	local Nusselt number, $h(z)D_h/K_f$
Pe	=	Peclet number, $\bar{u}D_h/\alpha$
R, r	=	radial coordinate, dimensionless and dimensional, respectively
r_1, r_2	=	radius of duct wall, internal and external, respectively
T_e, T_∞	=	inlet and ambient temperatures, respectively
$T_f(r, z)$, $T_s(r, z)$	=	fluid and solid temperature distributions, respectively
$\bar{u}$	=	average flow velocity
$u(r)$, $U(R)$	=	velocity distribution, dimensional and dimensionless, respectively
Z, z	=	axial coordinate, dimensionless and dimensional, respectively
α	=	thermal diffusivity of the fluid
β	=	conjugation parameter (dimensionless)
δ	=	aspect ratio of circular duct, r_2/r_1
$\theta(R, Z)$	=	dimensionless fluid temperature distribution
μ_i	=	eigenvalues of Sturm-Liouville problem
$\psi_i(R)$	=	eigenfunctions of Sturm-Liouville problem

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Introduction

LAMINAR forced convection inside ducts is usually studied by neglecting the participation of the tube wall in the heat transfer process through imposition of temperature or heat flux, as in the classical Graetz problem,[1] at the fluid-solid interface. As pointed out by recent reviews,[1,2] the inclusion of wall conduction effects is of major relevance to the accurate prediction of heat transfer rates, but brings up a conjugated conduction-convection problem of a more involved nature when the full energy equations for both fluid and solid are to be solved simultaneously. Early work on approximate analytical solutions[3,4] demonstrates the mathematical difficulties involved. More recently, purely numerical approaches[5,6] illustrate the high computational costs for the accurate solution of such coupled conduction-convection problems, governed by at least four parameters, namely, Peclet number, aspect ratio, Biot number, and a wall-to-fluid conductance parameter. However, a simpler model was proposed[1] that radially lumped the temperature distribution at the tube wall, but retained the axial conduction information along the wall, thereby reducing the number of parameters to be explicitly considered. The simplified model was expected to be particularly useful in reducing computational costs and analytical involvement, aspects not fully achieved by previous works in this direction.[7-9] The lumped formulation, although expected to be more adequate in the range of parameters that provides not so significant radial temperature gradients within the wall, has been checked only briefly against numerical solutions that consider the two-dimensional effects.[6,10] The present work brings the analytical solution to this class of problems through extension of the ideas in the so-called generalized integral transform technique[11-18] in order to allow for the handling of the more general type of boundary conditions that results from the lumping procedure at the wall. Accurate numerical results provide a set of benchmark results for quantities of major interest in heat exchangers theory, such as bulk temperatures and Nusselt numbers. The effects of Biot number and of a conjugation parameter on heat transfer rates and temperature distributions are then investigated.

Analysis

Laminar forced convection within the thermal entrance region of a circular duct is considered, according to Fig. 1, for the flow of a Newtonian fluid under constant properties and negligible viscous dissipation, free convection, and fluid axial conduction effects. On the other hand, the duct wall is allowed to influence the heat transfer process characterizing the so-

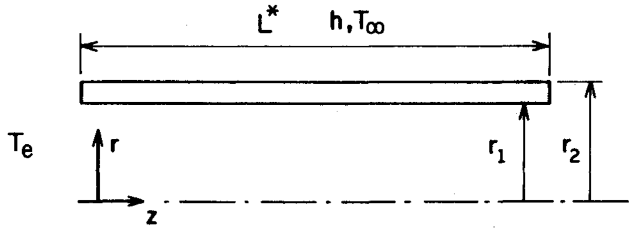


Fig. 1 Geometry and coordinate system for conjugated problem in a circular tube.

called conjugated heat transfer problem,[3] which is written in dimensionless form as

Solid region

$$\frac{\partial^2\theta_s(R,Z)}{\partial Z^2} + \frac{4\,Pe^2}{R}\frac{\partial}{\partial R}\left[R\frac{\partial\theta_s(R,Z)}{\partial R}\right] = 0$$

$$1 < R < \delta,\ 0 < Z < L \tag{1a}$$

with boundary conditions

$$\frac{\partial\theta_s(R,0)}{\partial Z} = 0, \qquad 1 \le R \le \delta \tag{1b}$$

$$\frac{\partial\theta_s(R,L)}{\partial Z} = 0, \qquad 1 \le R \le \delta \tag{1c}$$

$$\frac{\partial\theta_s(\delta,Z)}{\partial R} + Bi^*\theta_s(\delta,Z) = 0, \qquad Z > 0 \tag{1d}$$

Fluid region

$$W(R)\frac{\partial\theta_f(R,Z)}{\partial Z} = \frac{\partial}{\partial R}\left[R\frac{\partial\theta_f(R,Z)}{\partial R}\right]$$

$$0 < R < 1,\ Z > 0 \tag{1e}$$

with inlet and boundary conditions

$$\theta_f(R,0) = 1, \qquad 0 \le R \le 1 \tag{1f}$$

$$\frac{\partial\theta_f(0,Z)}{\partial R} = 0, \qquad Z > 0 \tag{1g}$$

and the continuity conditions at the solid-fluid interface

$$\theta_f(1,Z) = \theta_s(1,Z), \qquad 0 \le Z \le L \tag{1h}$$

$$\hat{K}\frac{\partial\theta_f(1,Z)}{\partial R} = \frac{\partial\theta_s(1,Z)}{\partial R}, \qquad 0 \le Z \le L \tag{1i}$$

where various dimensionless groups are given by

$$R = \frac{r}{r_1}, \quad Z = \frac{\alpha_f z}{\bar{u}D_h^2}, \quad \delta = \frac{r_2}{r_1}$$

$$L = \frac{\alpha_f L^*}{\bar{u}D_h^2}, \quad \hat{K} = \frac{K_f}{K_s}, \quad Bi^* = \frac{hr_1}{K_s}$$

$$\theta(R,Z) = \frac{T(r,z) - T_\infty}{T_e - T_\infty}, \quad U(R) = \frac{u(r)}{\bar{u}}$$

$$= 2(1 - R^2) \qquad W(R) = \frac{r_1^2}{D_h^2}RU(R) \tag{2}$$

The analytical solution of the complete system (1) is, indeed, a quite involved matter, as discussed in Ref. 3, and, here, a simplified model first proposed in Ref. 1 is employed that takes into account the axial conduction along the wall but assumes that the temperature distribution across the solid region can be lumped, as shown in the following. The energy equation for the duct wall is operated on with

$$\frac{1}{\pi(\delta^2 - 1)}\int_1^\delta 2\pi R\,\mathrm{d}R$$

to yield:

$$\frac{\mathrm{d}^2\theta_{s,m}(Z)}{\mathrm{d}Z^2} + \frac{8Pe^2}{\delta^2 - 1}\left[\frac{\delta\partial\theta_s(\delta,Z)}{\partial R} - \frac{\partial\theta_s(1,Z)}{\partial R}\right] = 0 \tag{3a}$$

where the average wall temperature is defined as

$$\theta_{s,m}(Z) = \frac{2}{\delta^2 - 1}\int_1^\delta R\theta_s(R,Z)\,\mathrm{d}R \tag{3b}$$

The boundary and interface conditions, Eqs. (1d) and (1i), respectively, are substituted into Eq. (3a) to provide the following

$$\frac{\mathrm{d}^2\theta_{s,m}(Z)}{\mathrm{d}Z^2} - \frac{8Pe^2}{\delta^2 - 1}\left[\delta Bi^*\theta_s(\delta,Z) + \hat{K}\frac{\partial\theta_f(1,Z)}{\partial R}\right] = 0 \tag{3c}$$

Under conditions of negligible transversal temperature gradients within the wall, the thermally thin wall approximation can be invoked, i.e.

$$\theta_{s,m}(Z) \simeq \theta_s(\delta,Z) \simeq \theta_s(1,Z) \tag{4}$$

and by recalling the interface condition, Eq. (1h), one obtains

$$\beta\frac{\partial^2\theta_f(1,Z)}{\partial Z^2} = \frac{\partial\theta_f(1,Z)}{\partial R} + Bi\theta_f(1,Z) \tag{5a}$$

where

$$Bi = \frac{\delta Bi^*}{\hat{K}} \tag{5b}$$

$$\beta = \frac{(\delta^2 - 1)}{8Pe^2\hat{K}} \tag{5c}$$

Therefore, the original temperature problem for the wall is transformed into a more general boundary condition at the fluid-solid interface ($R = 1$) for the fluid temperature problem. Also, the new conjugation parameter β incorporates the effects of the three parameters that govern the complete conjugated problem, namely, Peclet number Pe, thermal conductivities ratio $\hat{K}$, and aspect ratio δ, which markedly reduces the computational effort when seeking numerical results for different values of such parameters.

Equations (1e–g), together with Eq. (5a), form an extended Graetz-type problem that includes a fin effect at the boundary due to axial conduction along the wall and an effective thermal resistance for heat flow to the ambient characterized by an effective Biot number Bi. An analytical solution to this class of problems is pursued through extension of the ideas in the so-called generalized integral transform technique,[11–18] as now shown. The appropriate auxiliary problem is taken as

$$\frac{\mathrm{d}}{\mathrm{d}R}\left[R\frac{\mathrm{d}\psi_i(R)}{\mathrm{d}R}\right] + \mu_i^2W(R)\psi_i(R) = 0, \qquad 0 < R < 1 \tag{6a}$$

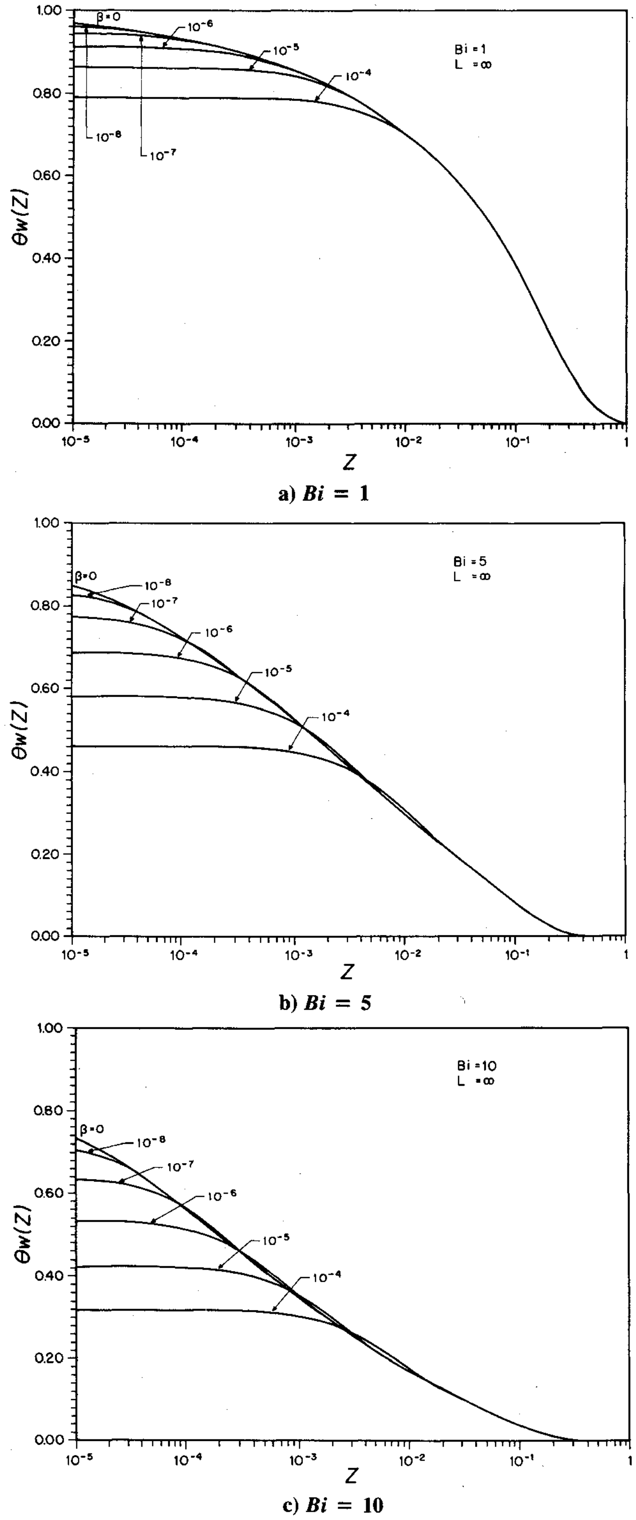


Fig. 2 Wall temperature distributions for different values of the conjugation parameter β.

with boundary conditions

$$\frac{d\psi_i(0)}{dR} = 0 \tag{6b}$$

$$\frac{d\psi_i(1)}{dR} + Bi\psi_i(1) = 0 \tag{6c}$$

whose solution for the related eigenvalues μ_i's and eigenfunctions ψ_i's is at this point assumed to be known through application of the recently advanced sign-count method.[19]

Equations (6) allow the definition of the following integral transform pair:

Transform

$$\bar{\theta}_i(Z) = \int_0^1 W(R)\frac{\psi_i(R)}{N_i^{1/2}}\theta_f(R, Z)\,dR \tag{7a}$$

Inverse

$$\theta_f(R, Z) = \sum_{i=1}^{\infty} \frac{\psi_i(R)}{N_i^{1/2}}\bar{\theta}_i(Z) \tag{7b}$$

where the normalization integral is given by

$$N_i = \int_0^1 W(R)\psi_i^2(R)\,dR \tag{7c}$$

Following the formalism in the generalized integral transform technique, Eq. (1e) is now operated on with

$$\int_0^1 \frac{\psi_i(R)}{N_i^{1/2}}\,dR$$

to yield

$$\frac{d\bar{\theta}_i(Z)}{dZ} + \mu_i^2\bar{\theta}_i(Z) = \frac{1}{N_i^{1/2}}\left[\psi_i(1)\frac{\partial\theta_f(1, Z)}{\partial R} - \theta_f(1, Z)\frac{\partial\psi_i(1)}{\partial R}\right] \tag{8a}$$

The right side of Eq. (8a) is evaluated through manipulation of the boundary conditions, Eqs. (5a) and (6c), to provide the following

$$\psi_i(1)\frac{\partial\theta_f(1, Z)}{\partial R} - \theta_f(1, Z)\frac{\partial\psi_i(1)}{\partial R} = \beta\psi_i(1)\frac{\partial^2\theta_f(1, Z)}{\partial Z^2} \tag{8b}$$

and the transformed equation becomes

$$\frac{d\bar{\theta}_i(Z)}{dZ} + \mu_i^2\bar{\theta}_i(Z) = \beta\frac{\psi_i(1)}{N_i^{1/2}}\frac{\partial^2\theta_f(1, Z)}{\partial Z^2}, \qquad i = 1, 2, \ldots \tag{8c}$$

On the other hand, the derivative with respect to R in Eq. (5a) can be expressed in terms of the transformed potentials, $\bar{\theta}_i(Z)$, by recalling the energy integral balance within the fluid obtained through integration of Eq. (1e)

$$\frac{\partial\theta_f(1, Z)}{\partial R} = \frac{1}{8}\frac{d\theta_{av}(Z)}{dZ} \tag{9a}$$

where the fluid bulk temperature is defined as

$$\theta_{av}(Z) = 8\int_0^1 W(R)\theta_f(R, Z)\,dR \tag{9b}$$

Or, upon substitution of the inverse formula, Eq. (7b)

$$\theta_{av}(Z) = 8\sum_{i=1}^{\infty} \bar{f}_i\bar{\theta}_i(Z) \tag{9c}$$

with

$$\bar{f}_i = \int_0^1 W(R)\frac{\psi_i(R)}{N_i^{1/2}}\,dR \tag{9d}$$

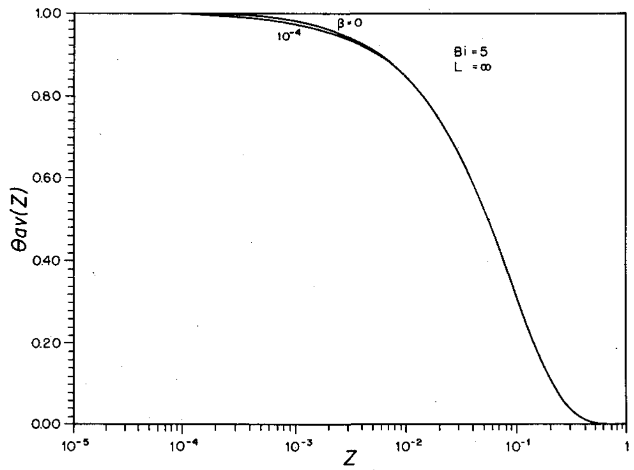


Fig. 3 Average fluid temperature distributions for different values of the conjugation parameter β: $Bi = 5$.

The boundary equation (5a) then becomes

$$\beta \frac{\mathrm{d}^2\theta_w(Z)}{\mathrm{d}Z^2} - Bi\theta_w(Z) = \sum_{j=1}^{\infty} \bar{f}_j \frac{\mathrm{d}\bar{\theta}_j(Z)}{\mathrm{d}Z} \tag{10}$$

where, for convenience, we write $\theta_w(Z) \equiv \theta_f(1, Z)$.

In deriving Eq. (10), the direct substitution of the inverse formula (7b) into the radial derivative at $R = 1$ was avoided. Since the boundary condition for the original problem, Eq. (5a), and that for the eigenvalue problem, Eq. (6c), are not exactly of the same form, the evaluation of quantities at this boundary through the inversion would involve a slowly converging series that could bring up undesirable errors in the realm of computations. The alternative expression based on the axial variation of the bulk temperature, Eq. (9a), has a much enhanced convergence behavior, as shall be illustrated later.

Equations (8c) and (10) are to be solved simultaneously, subjected to the transformed inlet and boundary conditions

$$\bar{\theta}_i(0) = \bar{f}_i, \qquad i = 1, 2, \ldots \tag{11a}$$

$$\frac{\mathrm{d}\theta_w(0)}{\mathrm{d}Z} = 0 \tag{11b}$$

$$\frac{\mathrm{d}\theta_w(L)}{\mathrm{d}Z} = 0 \tag{11c}$$

once these equations are rewritten in normal form, as now shown. First, the second derivative term in Eq. (8c) is eliminated by direct substitution of Eq. (10), to yield

$$\sum_{j=1}^{\infty} a_{ij} \frac{\mathrm{d}\bar{\theta}_j(Z)}{\mathrm{d}Z} + \mu_i^2\bar{\theta}_i(Z) = \frac{\psi_i(1)}{N_i^{1/2}} Bi\theta_w(Z) \tag{12a}$$

Or in matrix form

$$A\bar{\boldsymbol{\theta}}'(Z) + D\bar{\boldsymbol{\theta}}(Z) = \theta_w(Z)\boldsymbol{g} \tag{12b}$$

where

$$A = \{a_{ij}\}, \qquad a_{ij} = \delta_{ij} - \frac{\psi_i(1)}{N_i^{1/2}} \bar{f}_j \tag{12c}$$

$$D = \{d_{ij}\}, \qquad d_{ij} = \delta_{ij}\mu_i^2 \tag{12d}$$

$$\boldsymbol{g} = \{g_i\}^T, \qquad g_i = \frac{\psi_i(1)}{N_i^{1/2}} Bi \tag{12e}$$

with $\delta_{ij} = 0$, for $i \neq j$, and $\delta_{ij} = 1$, for $i = j$.

System (12b) is operated on with the inverse of the coefficients matrix A^{-1} to provide the equivalent system in normal form

$$\frac{\mathrm{d}\bar{\theta}_i(Z)}{\mathrm{d}Z} + \sum_{j=1}^{\infty} e_{ij}\bar{\theta}_j(Z) = h_i\theta_w(Z), \qquad i = 1, 2, \ldots \tag{13a}$$

where

$$E = \{e_{ij}\}, \qquad E = A^{-1}D \tag{13b}$$

$$\boldsymbol{h} = \{h_i\}^T, \qquad \boldsymbol{h} = A^{-1}\boldsymbol{g} \tag{13c}$$

Similarly, Eq. (13a) is directly substituted into the differential equation for the wall temperature, Eq. (10), providing the alternative form

$$\beta \frac{\mathrm{d}^2\theta_w(Z)}{\mathrm{d}Z^2} - \left(Bi + \sum_{j=1}^{\infty} \bar{f}_j h_j\right) \theta_w(Z) = -\sum_{j=1}^{\infty} \left(\sum_{k=1}^{\infty} \bar{f}_k e_{kj}\right) \bar{\theta}_j(Z) \tag{14}$$

Equations (13a) and (14) can now be brought together in the single first-order system once the infinite sums are truncated at a sufficiently large order N, as

$$\boldsymbol{y}'(Z) = B\boldsymbol{y}(Z) \tag{15a}$$

where

$$\boldsymbol{y} = \{\bar{\theta}_1(Z), \ldots, \bar{\theta}_N(Z), \theta_w(Z), \theta_w'(Z)\}^T \tag{15b}$$

and the $(N + 2) \times (N + 2)$ elements of matrix B are obtained from

$$B = \{b_{ij}\} = \begin{cases} -e_{ij}, \; i \leq N, j \leq N \\ h_i, \; i \leq N, j = N + 1 \\ 0, \; i \leq N, j = N + 2 \\ \delta_{j,N+2}, \; i = N + 1, j \leq N + 2 \\ -\dfrac{1}{\beta}\left[\displaystyle\sum_{k=1}^{N} \bar{f}_k e_{kj}\right] \\ \quad i = N + 2, j \leq N \\ \dfrac{1}{\beta}\left[Bi + \displaystyle\sum_{k=1}^{N} \bar{f}_k h_k\right] \\ \quad i = N + 2, j = N + 1 \\ 0, \; i = N + 2, j = N + 2 \end{cases} \tag{15c}$$

The solution vector $\boldsymbol{y}(Z)$ is explicitly obtained in terms of the eigenvalues and eigenvectors of matrix B, provided the following algebraic problem is solved

$$(B - \lambda I)\,\boldsymbol{\xi} = 0 \tag{16a}$$

for the corresponding eigenvalues λ and eigenvectors $\boldsymbol{\xi}$. The components of the solution vector are then given by

$$\boldsymbol{y}(Z) = \sum_{k=1}^{N+2} C_k \boldsymbol{\xi}^{(k)} e^{\lambda_k Z} \tag{16b}$$

where the constants C_k are obtained from satisfaction of the inlet and boundary conditions, Eqs. (11a–c), which results in the following linear system of algebraic equations

$$\sum_{k=1}^{N+2} C_k \xi_j^{(k)} = \bar{f}_j, \qquad \text{for } j = 1, 2, \ldots, N$$

$$\sum_{k=1}^{N+2} C_k \lambda_k \xi_{N+1}^{(k)} = 0$$

$$\sum_{k=1}^{N+2} C_k \lambda_k \xi_{N+1}^{(k)} e^{\lambda_k L} = 0 \tag{16c}$$

Well-established algorithms are readily available in advanced scientific subroutines packages, such as the IMSL library,[20] to solve both Eqs. (16a) and (16c). Once the transformed potentials are obtained from the solution vector, the inverse formula, Eq. (7b), can be recalled to construct the original fluid temperature field, whereas the lumped wall temperature distribution is directly obtained from the $(N + 1)$th element of $\boldsymbol{y}$.

Also of interest is the evaluation of local Nusselt numbers, obtained from the definition

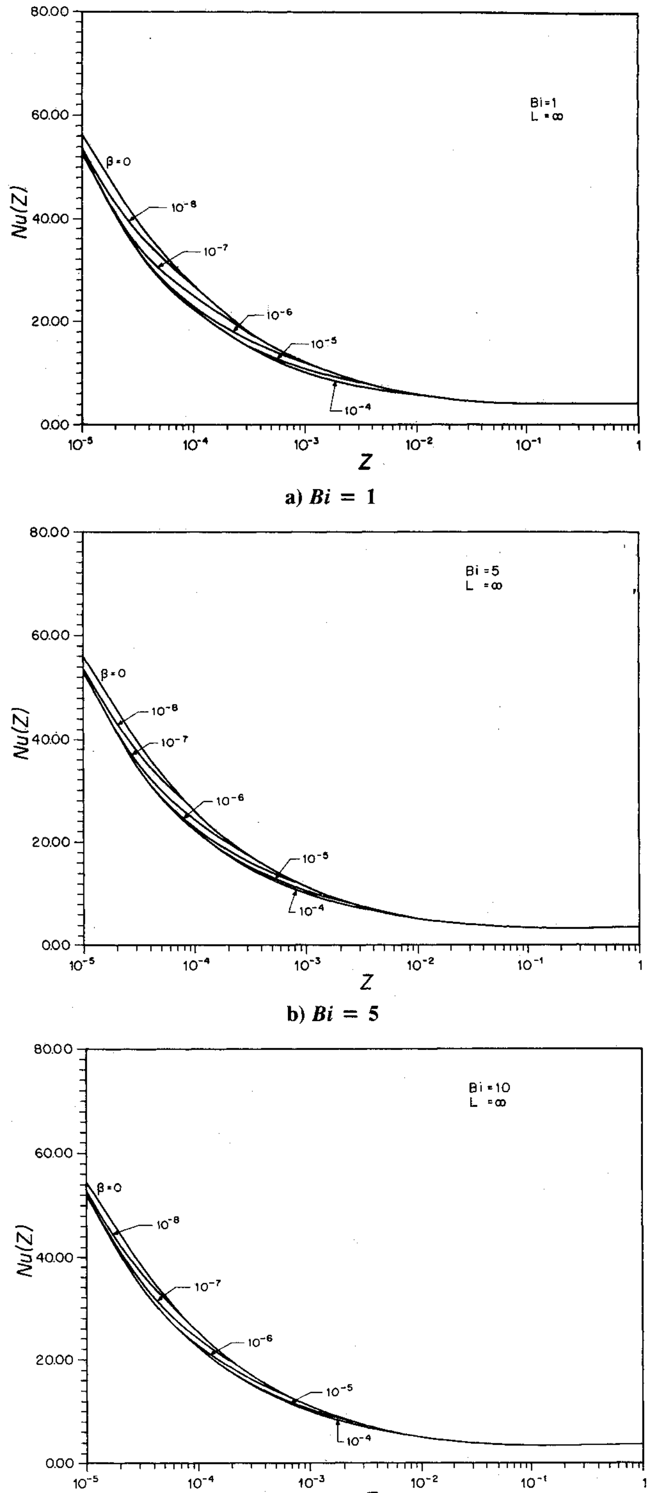
$$Nu(Z) = \frac{h(Z)D_h}{K_f} = \frac{-2[\partial\theta_f(1, Z)/\partial R]}{\theta_{av}(Z) - \theta_w(Z)} \tag{17}$$

Results and Discussion

Numerical results were obtained to allow investigation of the effects on heat transfer of the conjugation parameter β and effective Biot number Bi by taking the truncation at $N \leq 60$ terms to ensure convergence within an ample range of the dimensionless axial distance, $10^{-5} \leq Z \leq 10^{0}$. From practical considerations, typical values of the parameters were taken as $\beta = 10^{-4}$, 10^{-5}, 10^{-6}, 10^{-7} and 10^{-8}, and $Bi = 1$, 5, and 10. The case of $\beta = 0$ corresponds to the classical Graetz problem without wall conjugation effects, but with a third kind boundary condition.

Figures 2a–c show the lumped wall temperatures along the duct length for the representative values of β and Bi and for an infinite tube. For increasing β, when axial conduction along the wall becomes more important, the temperature distribution is progressively flattened up and a limiting value is attained in the region close to the inlet, where the deviations from the special situation of a nonparticipating wall ($\beta = 0$) are most noticeable. In fact, for the other limiting situation of $\beta \to \infty$, the Graetz problem with first kind boundary condition is recovered, which corresponds to a uniform prescribed temperature at the wall. This tendency to flatten up the wall temperature distributions is even more noticeable when the effective Biot number increases, again since the limiting case of $Bi \to \infty$ corresponds to a uniform prescribed temperature. For sufficiently large axial distances, the temperature gradients along the wall become small enough for the wall conjugation effect to disappear and the curves for different values of β end up by joining the curve for the Graetz problem with third kind boundary condition only. Therefore, the fin effect due to the wall conduction tends to smooth out the temperature distribution in the regions close to the inlet by diffusing heat away to regions of less significant gradients. Additional numerical results were obtained and critically compared to those in Ref. 9 for the wall temperature distributions ($\beta = 2.5 \times 10^{-3}$ and 2.5×10^{-4}; $Bi = 10$), showing an almost perfect agreement in the full range of the dimensionless axial distance.

The effects on fluid bulk temperatures of the conjugation parameter β are shown in Fig. 3. Clearly, only for the value $\beta = 10^{-4}$ some deviations are noticeable to the present graph scale, indicating that only the fluid in the regions close to the duct wall are markedly affected by the changes in wall temperature but, on the other hand, such layers have a minor contribution to the bulk temperature result. The same trends are observed for different values of the Biot number.



a) $Bi = 1$

b) $Bi = 5$

c) $Bi = 10$

Fig. 4 Local Nusselt numbers for different values of the conjugation parameter β.

The local Nusselt numbers are presented in Figures 4a–c, for different values of β and Bi, in the range considered. As β increases, and also the axial heat conduction within the wall, the curves in the region close to the inlet tend to join the prescribed temperature solution of the Graetz problem since the wall temperature is, in fact, becoming closer to uniform as β increases and the bulk temperature is not very

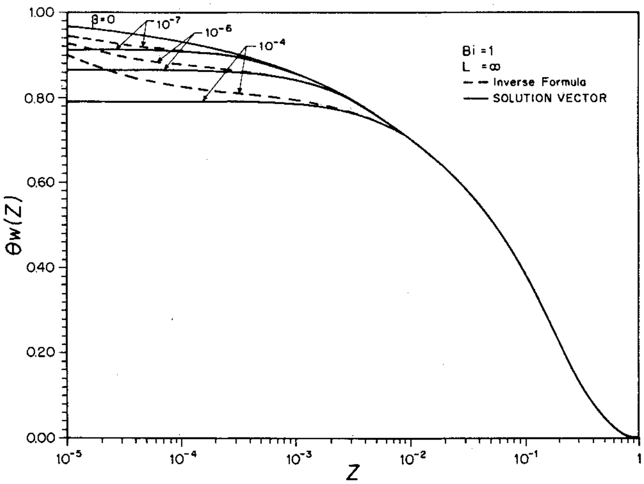


Fig. 5 Effects of the inverse formula applied at the boundary R = 1 on convergence behavior of the wall temperature.

much affected. For sufficiently large axial distances, again the curves join the Graetz problem solution with third kind boundary condition, which corresponds to negligible axial wall conduction. Therefore, these two limiting situations completely encapsulate the behavior of the local Nusselt number. For increasing Bi, the curves altogether become closer to the prescribed temperature situation since the two limiting curves approach each other.

Also of interest is the analysis of convergence behavior for the inverse formula, Eq. (7b), when applied to the boundary $R = 1$. As mentioned in the analysis section, the plain substitution of Eq. (7b) for the wall temperature, $\theta_w(Z)$, in Eq. (8b) could introduce some error due to the fact that the boundary conditions of the original P.D.E. and of the auxiliary eigenvalue problem were not identical; thus, the eigenfunction expansion at $R = 1$ represented by Eq. (7b) could be slowly convergent. Then, the transformed P.D.E. and the original boundary condition were manipulated to yield the rapidly converging system (11) here solved. Figure 5 now confirms the need for the present approach by presenting the wall temperature profiles as computed from the inverse formula (7b), compared against the fully converged solution from Eq. (10b). As the value of β is increased, and for shorter axial distances, the inverse formula gradually deviates from the exact solution for a finite number of terms and convergence becomes prohibitively slow.

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